Gravitation

- Newton's law of universal gravitation states that the gravitational force of attractionbetween any two particles of masses *m1* and *m2* separated by a distance *r* has themagnitude

  $F = Gm1m2/r^2$

  where G is the universal gravitational constant, which has the value 6.672 ×10–11 N m2 kg^{-2}.

- If we have to find the resultant gravitational force acting on the particle m due to anumber of masses M1, M2,….Mn etc. we use the principle of superposition. Let F1, F2,….Fnbe the individual forces due to M1, M2,….Mn, each given by the law of gravitation. Fromthe principle of superposition each force acts independently and uninfluenced by theother bodies. The resultant force FR is then found by vector addition

  FR = F1 + F2 +……+ Fn

- Kepler’s laws of planetary motion state that

  (a) All planets move in elliptical orbits with the Sun at one of the focal points

  (b) The radius vector drawn from the sun to a planet sweeps out equal areas in equaltime intervals. This follows from the fact that the force of gravitation on the planet iscentral and hence angular momentum is conserved.

  (c) The square of the orbital period of a planet is proportional to the cube of the semi-majoraxis of the elliptical orbit of the planet

  The period T and radius R of the circular orbit of a planet about the Sun are relatedby

  $T^2 = [4\pi^2/GMs]R^3$

  where Ms is the mass of the Sun. Most planets have nearly circular orbits about the Sun. For elliptical orbits, the above equation is valid if R is replaced by the semi-major axis, a.

- The acceleration due to gravity.

  at a height h above the Earth’s surface

  $g(h) = GM_e/(R_e + h)^2$

  $= GM_e/R_e^2\ [1 – (2h/R_e)]$ for $h << R_e$

at depth d below the Earth's surface is

$g(d) = GM_e/R_e^2 [1 – (d/R_e)]$

- The gravitational force is a conservative force, and therefore a potential energy function can be defined. The gravitational potential energy associated with two particles separated by a distance r is given by

  V = -Gm1m2/r

  where V is taken to be zero at $r \to \infty$. The total potential energy for a system of particles is the sum of energies for all pairs of particles, with each pair represented by a term of the form given by above equation. This prescription follows from the principle of superposition.

- If an isolated system consists of a particle of mass m moving with a speed v in thevicinity of a massive body of mass M, the total mechanical energy of the particle is given by

  $E = (1/2)mv^2 – [GMm/r]$

  That is, the total mechanical energy is the sum of the kinetic and potential energies. The total energy is a constant of motion.

- If m moves in a circular orbit of radius a about M, where M >>m, the total energy of thesystem is

  $E = - GMm/2a$
- The escape speed from the surface of the Earth is $\sqrt{2gR_E}$

  and has a value of 11.2 km s^{-1}.
- If a particle is outside a uniform spherical shell or solid sphere with a sphericallysymmetric internal mass distribution, the sphere attracts the particle as though themass of the sphere or shell were concentrated at the centre of the sphere.
- If a particle is inside a uniform spherical shell, the gravitational force on the particle iszero. If a particle is inside a homogeneous solid sphere, the force on the particle actstoward the centre of the sphere. This force is exerted by the spherical mass interior tothe particle.
- A geostationary (geosynchronous communication) satellite moves in a circular orbit inthe equatorial plane at a approximate distance of 4.22×10^4 km from the Earth's centre.

Sample Examples

- A 400 kg satellite is in acircular orbit of radius $2R_E$ about theEarth. How much energy is required totransfer it to a circular orbit of radius $4R_E$?What are the changes in the kinetic andpotential energies ?

  Solution

  Initially,

  $Ei = - GM_e m/4R_E$

  Finally,

  $Ef = - GM_e m/8R_E$
  $\Delta E = Ef – Ei = gmR_E/8$

  $= 3.13\ 10^9$ J

  The kinetic energy is reduced and it mimics ΔE, namely, $\Delta K = Kf – Ki = – 3.13 \times 10^9$ J.

  The change in potential energy is twice the change in the total energy, namely

  $\Delta V = Vf – Vi = – 6.25 \times 10^9$ J

- The planet Mars has twomoons, phobos and delmos. (i) phobos hasa period 7 hours, 39 minutes and an orbital radius of 9.4×10^3 km. Calculate the massof mars. (ii) Assume that earth and marsmove in circular orbits around the sun,with the martian orbit being 1.52 timesthe orbital radius of the earth. What isthe length of the martian year in days?

  Solution

  $T^2 = (4\pi^2R^3/GM_m)$

  $M_m = (4\pi^2R^3/GT^2) = [\ (4*(3.14)^2 * (9.4)^3 * 10^{18})/6.67 * (4.59*6)^2 *10^{-5})]$

  $=6.48 \times 10^{23}$ kg.

  $T_M{}^2/T_E{}^2 = R_{MS}{}^3/R_{ES}{}^3$

  where $R_{MS}$is the mars -sun distance and $R_{ES}$is the earth-sun distance.

  $\therefore T_M = (1.52)^{3/2} \times 365 = 684$ days